SECTION: PHYSICAL AND MATHEMATICAL SCIENCES

DOI 10.70286/ISU-01.04.2026.017

METHOD OF APPROXIMATING THE ROOTS OF NONLINEAR EQUATIONS WITH AN ADAPTIVE STEP

Doroshenko Daniil
Faculty of Mechanics and Mathematics
Oles Honchar Dnipro National University, Ukraine

In many problems of mathematics, physics and engineering there is a need to find the roots of nonlinear equations. Analytical solution of such equations is often difficult or impossible, therefore in practice methods of approximate calculations are used. These methods allow to obtain a solution with a given accuracy using iterative procedures.

However, each of the classical methods has its drawbacks. Some provide high accuracy, but may diverge, others are stable, but work slowly. In this regard, the search for approaches that combine the advantages of different methods is relevant.

Classical methods and their limitations. One of the most famous is Newton's method, which has a high convergence rate. Its iteration formula is as follows:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

This method is effective provided that the initial approximation is chosen well and the derivative of the function is not zero. However, in the case of complex behavior of the function, the method may lose convergence.

Another example is the bisection method, which guarantees finding the root on the segment, but converges more slowly. The chord method occupies an intermediate position between them in terms of speed and reliability.

Description of the hybrid approach. In order to increase the efficiency of calculations, a hybrid method is proposed that combines the advantages of several approaches. The main idea is that at each iteration the most appropriate calculation method is selected depending on the properties of the function.

If the value of the derivative is large enough and the function behaves smoothly, Newton's method is used. In cases where the derivative approaches zero or sharp changes in the function are observed, it is advisable to switch to a more stable method, for example, bisection.

To increase stability, an adaptive step coefficient is also introduced:

$$x_{n+1} = x_n - \alpha_n \frac{f(x_n)}{f'(x_n)}, 0 < \alpha_n \leq 1$$

The value of the coefficient α_n changes depending on the behavior of the function: in case of instability, it decreases, which allows avoiding process divergence.

Accuracy assessment. One of the important stages is to control the accuracy of the obtained result. For this, criteria for stopping the iterative process are used. The most common conditions are:

$$| x_{n+1} - x_n | < \varepsilon \quad \text{or} \quad | f(x_n) | < \varepsilon$$

where ε is the specified accuracy of the calculations.

Advantages of the proposed method. Let's consider the application of the method on a specific example. Let's find the root of the equation:

$$f(x) = x^3 - x - 2$$

This equation does not have a simple analytical solution, so numerical methods are used. When using Newton's method, the iterative process can quickly converge, but only if the initial approximation is chosen successfully. If the initial value is chosen unsuccessfully or the derivative approaches zero at some points, the process may become unstable.

The bisection method, on the contrary, is guaranteed to find the root on the segment, for example [1; 2], but requires a significantly larger number of iterations to achieve the same accuracy.

In the hybrid method, bisection is first used to locate the root, after which a transition is made to Newton's method with an adaptive coefficient. This allows you to combine the reliability of the initial stage with a high convergence rate in subsequent steps.

The proposed approach combines the convergence speed of Newton's method and the reliability of the bisection method. Thanks to the adaptive step selection, it is possible to reduce the risk of divergence and increase the accuracy of calculations. If the iteration becomes unstable at a certain step, the step factor is automatically reduced, which provides a smoother approximation to the root.

Practical evaluations show that the hybrid method provides stable results even in cases of complex function behavior, where classical methods work inefficiently. In particular, it is useful for functions with sharp changes or sections where the derivative changes sign.

Conclusions. The paper considers an approach to solving nonlinear equations, which is based on a combination of several methods of approximate calculations. The main idea is to use different algorithms depending on the behavior of the function at each stage of the calculation.

The proposed hybrid method allows to increase the efficiency of the computational process due to the adaptive choice of the algorithm and adjustment of the iteration step. This allows to simultaneously ensure both high accuracy and reliability of the obtained results.

Such an approach can be used in various applied problems, in particular in engineering calculations, physical models and computer simulation, where it is important to obtain an accurate result for a limited number of calculations. The prospect of further research is to improve the criteria for switching between methods and optimize the choice of the adaptive coefficient.

References

1. Burden R. L., Faires J. D. Numerical Analysis. 10th ed. Boston : Cengage Learning, 2016. 888 p. (https://doi.org/10.13140/2.1.4830.2406)
2. Sreedeep, C. D., Argyros, I. K., Gopika Dinesh, K. C., (2025). Convergence analysis of a power series based iterative method having seventh order of convergence. Math Studii, 64(2), 179-193.https://doi.org/10.30970/ms.64.2.179-193