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APPROXIMATION OF FUNCTIONS BY CHEBYSHOV INTERPOLATION POLYNOMIALS WITH INCREASED ACCURACY

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The problem of function approximation is one of the main ones in numerical analysis and is widely used in solving applied problems where the analytical form of the function is complex or unknown. In such cases, the function is replaced by a simpler approximating model, which is built on the basis of its values at a finite set of points.

Interpolation methods allow you to restore the function at given nodes, but the accuracy of the approximation depends significantly on their choice. With a uniform arrangement of nodes, significant fluctuations of the interpolation polynomial often occur at the edges of the segment, which leads to an increase in the error. In this regard, the use of special node systems that provide better convergence of the interpolation process is relevant.

The interpolation polynomial $P_n(x)$ is constructed so that the condition is met:

$$P_n(x_k) = f(x_k), k = 0, 1, \dots, n.$$

One of the most common forms is the Lagrange interpolation polynomial:

$$P_n(x) = \sum_{k=0}^{n} f(x_k) \cdot L_k(x)$$

where the basis polynomials are defined as:

$$L_k(x) = \prod_{j=0, j\neq k}^{n} \frac{x - x_j}{x_k - x_j}$$

The accuracy of interpolation largely depends on the choice of the x_k nodes. To reduce the error, it is advisable to use Chebyshev nodes, which are on the segment [-1; 1] are given by the formula:

$$x_k = \cos\frac{(2k+1)\cdot\pi}{2n+2}, k = 0,1, \dots n.$$

For an arbitrary segment [a, b], the transformation is used:

$$x_k = \frac{a+b}{2} + \frac{b-a}{2} \cdot x_k$$

The uneven distribution of Chebyshev nodes allows to reduce the influence of edge effects and provides a more uniform distribution of the error over the entire segment. It is thanks to this that the interpolation polynomial demonstrates better behavior compared to the case of uniform selection of nodes, where significant oscillations are possible [1, p.303]

The interpolation error is given by the formula:

$$R_n(x) = f(x) - P_n(x) = \frac{f^{n+1}(\xi)}{(n+1)!} \cdot \prod_{k=0}^{n} (x - x_k)$$

where $\xi \in [a, b]$. From this formula it is clear that it is the factor $\prod(x - x_k)$ that determines the sensitivity of the error to the choice of nodes, therefore their optimal location is crucial for the accuracy of the approximation.

Example. Consider the function: $f(x) = \frac{1}{1+25x^2}, x \in [-1, 1]$

Let n = 4. Chebyshev nodes are defined as:

$$x_k = \cos\frac{(2k+1) \cdot \pi}{10}, k = 0,1,2,3,4$$

Approximate node values:

$$x_0 \approx 0.9511$$

$$\dots$$

$$x_4 \approx -0.9511$$

Function values at nodes:

$$f(0) = 1$$

$$f(\pm 0.9511) \approx 0.042$$

Interpolation polynomial:

$$P_4(x) = \sum_{k=0}^{4} f(x_k) \cdot L_k(x)$$

The obtained approximation shows stable behavior over the entire segment and the absence of significant oscillations at the edges, which is typical for the case of uniform node placement. This confirms the effectiveness of using Chebyshev nodes to improve the accuracy of interpolation.

Conclusion. The paper considers an approach to approximating functions using interpolation polynomials using Chebyshev nodes. It is shown that the choice of interpolation nodes significantly affects the accuracy and stability of calculations.

The use of Chebyshev nodes allows you to reduce the maximum interpolation error, avoid oscillations and ensure a more uniform convergence of the polynomial. Thus, optimizing the location of the nodes is an effective way to improve the quality of function approximation without complicating the interpolation scheme itself.

References

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DEVELOPING STUDENTS' COMPUTATIONAL SKILLS IN MATHEMATICS IN DISTANCE EDUCATION CONTEXTS

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Introduction. Distance learning is no longer seen as a temporary solution but has become a regular part of teaching practice, including in mathematics classes. At the same time, it has made it clear that not all learning processes transfer easily to the online format. This is especially noticeable when it comes to developing basic skills such as computation, which usually requires consistent practice and immediate feedback from the teacher.

For learners aged around 10–12, computational skills involve more than simply getting correct answers. They include understanding how numbers relate to each other, following step-by-step procedures, and working accurately with multi-step tasks. In a traditional classroom, these skills are supported through regular explanation, guided practice, and timely correction. In distance learning, this support becomes less direct and sometimes inconsistent.

As a result, students may complete tasks mechanically or have difficulties noticing and correcting their own mistakes. Maintaining attention and consistency in learning also becomes more challenging. This shows that teaching mathematics online requires a more carefully structured approach.

The aim of this study is to identify effective ways of developing students' computational skills in mathematics in distance education and to examine their impact on learning outcomes.

Literature review. Research in recent years increasingly shows that computational skills in mathematics cannot be reduced to simple repetition of exercises. As noted in the review by Fyfe et al. (2024), feedback in mathematics works only when students actually engage with it and understand how it relates to their mistakes. A similar point is made by Hershkovitz et al. (2024), who found that in digital environments many learners tend to ignore feedback, which leads to repeated errors. This becomes especially problematic in computational tasks, where misunderstanding of one step often affects the entire solution.