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LINEARLY INDEPENDENT BASIS OF HYPERCOMPLEX NUMBERS BASED ON COMBINATIONS OF REAL, IMAGINARY, DOUBLE, AND DUAL NUMBERS (FINSLERIAN METRIC)

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This paper continues the study of hypercomplex numbers based on combinations of real, imaginary, double, and dual numbers.

There is considered number of the form: $w = a + ib + \gamma c + i\gamma d$, where $i^2 = -1$, $\gamma^2 = 1$. The addition and multiplication functions are defined as follows:

$$w_1 + w_2 = a_1 + a_2 + i(b_1 + b_2) + \gamma(c_1 + c_2) + i\gamma(d_1 + d_2),$$

$$w_1 w_x = a_1a_2 - b_1b_2 + c_1c_2 - d_1d_2 + i(a_1b_2 + b_1a_2 + c_1d_2 + d_1c_2) +$$
$$+ \gamma(a_1c_2 - b_1d_2 + c_1a_2 - d_1b_2) + i\gamma(a_1d_2 + b_1c_2 + c_1b_2 + d_1a_2),$$

This number can be represented as a matrix:

$$w = \begin{bmatrix} a & b & c & d \\ -b & a & -d & c \\ c & d & a & b \\ -d & c & -b & a \end{bmatrix}$$, and also in the exponential form:

$w = re^{i\varphi + \gamma\psi + i\gamma\varsigma}$, where $r = \sqrt[4]{\det(w)}$

$\det(w) = a^4 + b^4 + c^4 + d^4 + 2a^2b^2 - 2a^2c^2 + 2a^2d^2 + 2b^2c^2 - 2b^2d^2 + 2c^2d^2 - 8abcd$, - is the determinant of a matrix, the radius of a hypercomplex number, and one of the invariants of the algebra of the polynumber under consideration, whose metric is defined in terms of the fourth power of coordinates, indicating that the space under consideration, is Finslerian. The angles of the radius vector are determined by the following formulas:

$$\cos^2\varphi = \frac{(\det w)^{-1/2}(a^2 - b^2 - c^2 + d^2) + 1}{2}$$

$$ch^2\psi = \frac{(\det w)^{-1/2}(a^2 + b^2 + c^2 + d^2) + 1}{2}$$

$$\cos^2\varsigma = \frac{(\det w)^{-1/2}(a^2 + b^2 - c^2 - d^2) + 1}{2}$$

Formulas for determining the argument of a polycomplex number:

1) $w=a+ib+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{3}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}=re^{i\left(\varphi+\frac{1}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}$

2) $w=a+ib+\gamma c-i\gamma d=re^{i\varphi-\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)-\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

3) $w=a+ib-\gamma c+i\gamma d=re^{i\varphi+\gamma\psi+i\gamma\varsigma}=re^{i(\varphi+\pi)+\gamma\psi+i\gamma(\varsigma+\pi)}$

4) $w=a-ib+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{3}{2}\pi\right)-\gamma\psi+i\gamma\xi}=re^{i\left(\varphi+\frac{1}{2}\pi\right)-\gamma\psi+i\gamma(\varsigma+\pi)}$

5) $w=-a+ib+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{3}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}=re^{i\left(\varphi+\frac{1}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}$

6) $w=a+ib-\gamma c-i\gamma d=re^{i\left(\varphi+\frac{3}{2}\pi\right)-\gamma\psi+i\gamma(\varsigma+\pi)}=re^{i\left(\varphi+\frac{1}{2}\pi\right)-\gamma\psi+i\gamma\varsigma}$

7) $w=a-ib+\gamma c-i\gamma d=re^{i\varphi+\gamma\psi+i\gamma(\varsigma+\pi)}=re^{i(\varphi+\pi)+\gamma\psi+i\gamma\varsigma}$

8) $w=-a+ib+\gamma c-i\gamma d=re^{i\varphi-\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)-\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

9) $w=a+ib+\gamma c=re^{i\varphi+\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)+\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

10) $w=a+ib-\gamma c=re^{i\varphi-\gamma\psi+i\gamma\varsigma}=re^{i(\varphi+\pi)-\gamma\psi+i\gamma(\varsigma+\pi)}$

11) $w=a-ib+\gamma c=re^{i\left(\varphi+\frac{1}{2}\pi\right)+\gamma\psi+i\gamma(\varsigma+\pi)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)+\gamma\psi+i\gamma\varsigma}$

12) $w=-a+ib+\gamma c=re^{i\left(\varphi+\frac{1}{2}\pi\right)-\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)-\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

13) $w=a+ib+i\gamma d=re^{i\varphi+\gamma\psi+i\gamma\varsigma}=re^{i(\varphi+\pi)+\gamma\psi+i\gamma(\varsigma+\pi)}$

14) $w=a+ib-i\gamma d=re^{i\varphi-\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)-\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

15) $w=a-ib+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)-\gamma\psi+i\gamma(\varsigma+\pi)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)-\gamma\psi+i\gamma\varsigma}$

16) $w=-a+ib+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

17) $w=a+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)+\gamma\psi+i\gamma(\varsigma+\pi)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)+\gamma\psi+i\gamma\varsigma}$

18) $w=a+\gamma c-i\gamma d=re^{i\varphi+\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)+\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

19) $w=a-\gamma c+i\gamma d=re^{i\varphi-\gamma\psi+i\gamma\varsigma}=re^{i(\varphi+\pi)-\gamma\psi+i\gamma(\varsigma+\pi)}$

20) $w=-a+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)-\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)-\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

21) $w=ib+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)+\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

22) $w=ib+\gamma c-i\gamma d=re^{i\varphi-\gamma\psi+i\gamma\left(\varsigma+\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)-\gamma\psi+i\gamma\left(\varsigma+\frac{1}{2}\pi\right)}$

23) $w=ib-\gamma c+i\gamma d=re^{i\varphi+\gamma\psi+i\gamma\varsigma}=re^{i(\varphi+\pi)+\gamma\psi+i\gamma(\varsigma+\pi)}$

24) $w=-ib+\gamma c+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)-\gamma\psi+i\gamma(\varsigma+\pi)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)-\gamma\psi+i\gamma\varsigma}$

25) $w=\gamma c+i\gamma d=re^{i\left(\varphi+\frac{1}{2}\pi\right)+i\gamma\left(\frac{3}{2}\pi\right)}=re^{i\left(\varphi+\frac{3}{2}\pi\right)+i\gamma\left(\frac{1}{2}\pi\right)}$

26) $w=\gamma c-i\gamma d=re^{i\varphi+i\gamma\left(\frac{3}{2}\pi\right)}=re^{i(\varphi+\pi)+i\gamma\left(\frac{1}{2}\pi\right)}$

27) $w=ib+i\gamma d=re^{\gamma\psi+i\gamma\left(\frac{1}{2}\pi\right)}=re^{i\pi+\gamma\psi+i\gamma\left(\frac{3}{2}\pi\right)}$

28) $w = ib - i\gamma d = re^{i\pi - \gamma\psi + i\gamma\left(\frac{1}{2}\pi\right)} = re^{-\gamma\psi + i\gamma\left(\frac{3}{2}\pi\right)}$

29) $w = ib + \gamma c = re^{i\left(\frac{1}{2}\pi\right) + i\gamma\left(\varsigma + \frac{3}{2}\pi\right)} = re^{i\left(\frac{3}{2}\pi\right) + i\gamma\left(\varsigma + \frac{1}{2}\pi\right)}$

30) $w = ib - \gamma c = re^{i\left(\frac{1}{2}\pi\right) + i\gamma\varsigma} = re^{i\left(\frac{3}{2}\pi\right) + i\gamma(\varsigma + \pi)}$

31) $w = a + i\gamma d = re^{i\gamma\varsigma} = re^{i\pi + i\gamma(\varsigma + \pi)}$

32) $w = a - i\gamma d = re^{i\gamma\left(\varsigma + \frac{3}{2}\pi\right)} = re^{i\pi + i\gamma\left(\varsigma + \frac{1}{2}\pi\right)}$

33) $w = a + \gamma c = re^{i\left(\frac{1}{2}\pi\right) + \gamma\psi + i\gamma\left(\frac{3}{2}\pi\right)} = re^{i\left(\frac{3}{2}\pi\right) + \gamma\psi + i\gamma\left(\frac{1}{2}\pi\right)}$

34) $w = a - \gamma c = re^{i\left(\frac{1}{2}\pi\right) - \gamma\psi + i\gamma\left(\frac{1}{2}\pi\right)} = re^{i\left(\frac{3}{2}\pi\right) - \gamma\psi + i\gamma\left(\frac{3}{2}\pi\right)}$

35) $w = a + ib = re^{i\varphi} = re^{i(\varphi + \pi) + i\gamma\pi}$

36) $w = a - ib = re^{i\left(\varphi + \frac{3}{2}\pi\right)} = re^{i\left(\varphi + \frac{1}{2}\pi\right) + i\gamma\pi}$

37) $w = a = ae^{i\pi + i\gamma\pi}$

38) $w = ib = re^{i\left(\frac{1}{2}\pi\right)} = re^{i\left(\frac{3}{2}\pi\right) + i\gamma\pi}$

39) $w = \gamma c = re^{i\left(\frac{1}{2}\pi\right) + i\gamma\left(\frac{3}{2}\pi\right)} = re^{i\left(\frac{3}{2}\pi\right) + i\gamma\left(\frac{1}{2}\pi\right)}$

40) $w = i\gamma d = re^{i\gamma\left(\frac{1}{2}\pi\right)} = re^{i\pi + i\gamma\left(\frac{3}{2}\pi\right)}$

The other 40 variants of numbers are obtained from those considered by multiplying by $-1 = e^{i\pi}$.

Using the online program available at https://matrixcalc.org/ it was determined that the rank of the matrix w = 4, i.e., its basis is linearly independent. The transposed matrix w^T can be written as a polynumber:

$w^T = x_1 + x_2 i + x_3\gamma + x_4 i\gamma$, where $x_1 = (a^3 + ab^2 - ac^2 + ad^2 - 2bcd)/r^4$, $x_2 = (-b^3 - a^2 b - bc^2 + bd^2 + 2acd)/r^4$, $x_3 = (c^3 - a^2 c + b^2 c + cd^2 - 2abd)/r^4$, $x_4 = (-d^3 - a^2 d + b^2 d - c^2 d + 2abc)/r^4$.

Let's consider a polynumber of the form $z = x_1 + x_2 i + x_3\gamma + x_4 i\gamma + x_5\delta + x_6 i\delta + x_7\gamma\delta + x_8 i\gamma\delta$. It can be represented as a matrix:

$$z = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & -x_4 & x_3 & -x_6 & x_5 & -x_8 & x_7 \\ x_3 & x_4 & x_1 & x_2 & x_7 & x_8 & x_5 & x_6 \\ -x_4 & x_3 & -x_2 & x_1 & -x_8 & x_7 & -x_6 & x_5 \\ 0 & 0 & 0 & 0 & x_1 & x_2 & x_3 & x_4 \\ 0 & 0 & 0 & 0 & -x_2 & x_1 & -x_4 & x_3 \\ 0 & 0 & 0 & 0 & x_3 & x_4 & x_1 & x_2 \\ 0 & 0 & 0 & 0 & -x_4 & x_3 & -x_2 & x_1 \end{bmatrix}$$

$\det z = \sqrt[4]{\det w}$, where $a = x_1,\ b = x_2,\ c = x_3,\ d = x_4,$

The number z can be represented as $z = re^{i\varphi + \gamma\psi + i\gamma\varsigma + \delta D + i\delta H + \gamma\delta R + i\gamma\delta Q}$, where the coefficients D, H, Q, R are obtained by solving the following matrix equation:

$$\begin{bmatrix} x_1 & -x_2 & x_3 & -x_4 \\ x_2 & x_1 & x_4 & x_3 \\ x_3 & -x_4 & x_1 & -x_2 \\ x_4 & x_3 & x_2 & x_1 \end{bmatrix} \begin{bmatrix} D \\ H \\ Q \\ R \end{bmatrix} = \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

The rank of the matrix z = 8, i.e., its basis is linearly independent.

If $x_1 = \cos\varphi$, $x_2 = -\sin\varphi$, $x_3 = ch\psi$, $x_4 = sh\psi$, then we obtain the Givens matrix G, which determines the rotation of the vector w by a given angle φ and ψ: Gw.

In the work [5], I suggested that the proposed algebra of the polycomplex number z, constructed on the basis of combinations of real, imaginary, double, and dual numbers (its imaginary part), can be used as a mathematical apparatus for the "imaginary" geometry (the geometry of spiritual space where angels and the souls of the departed dwell), proposed by Pavel Florensky, in which the speed of propagation of interactions is greater than the speed of light and may also have another fundamental constant $v_{\max} > c$. Also in the work [5], a justification for the philosophical worldviews of the Orthodox religious concept is presented.

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